Mar. 22, 2017

Sect. 6-8

Partial Fraction Decomposition

Simplify:

$$\frac{1}{x-2} + \frac{2}{x+1} \qquad \text{LCD}: (x-2)(x+1)$$

$$= \frac{1(x+1)}{(x-2)(x+1)} + \frac{2(x-2)}{(x+1)(x-2)}$$

$$= \frac{3x-3}{x^2-x-2}$$

PFD goes the other way.

$$\frac{x+7}{x^2-x-6}$$

① $(x+2)(x-3)$

② $\frac{A}{x+2} + \frac{B}{x-3}$ LCD: $(x+2)(x-3)$

③ $\frac{A(x-3)}{(x+2)(x-3)} + \frac{B(x+2)}{(x-3)(x+2)}$

$$\frac{A(x-3)}{(x+2)(x-3)} + \frac{B(x+2)}{(x-3)(x+2)}$$

$$\frac{Ax - 3A + Bx + 2B}{x^2 - x - 6} = \frac{x+7}{x^2 - x - 6}$$

$$(A+B)x + (-3A+2B) = 1x + 7$$

$$\begin{cases} A+B=1 \\ -3A+2B=7 \end{cases} \Rightarrow A=1-B$$

$$-3(1-B)+2B=7$$
$$-3+3B+2B=7$$
$$5B=10$$
$$B=2$$

$$A=1-2$$
$$A=-1$$

Final Answer:

$$\frac{x+7}{x^2-x-6} = \frac{-1}{x+2} + \frac{2}{x-3}$$

Shortcut:

$$A(x-3) + B(x+2) = x+7$$

Pick Convienent x's

Let $x = 3$

$$A(3-3) + B(3+2) = 3+7$$

$$A(0) + B(5) = 10$$

$$5B = 10$$

$$B = 2$$

Let $x=-2$

$$A(-2-3)+B(-2+2)=-2+7$$

$$-5A=5$$

$$A=-1$$

$$\frac{x+7}{x^2-x-6}=\frac{-1}{x+2}+\frac{2}{x-3}$$

$$\frac{x^4+2x^3+6x^2+20x+6}{x^3+2x^2+x}$$

Long Div.

Remember me!

$$x + \frac{5x^2+20x+6}{x^3+2x^2+x}$$

Quot.

remainder

Do PFD on the remainder

$$\frac{5x^2+20x+6}{x^3+2x^2+x}$$

① $x(x^2+2x+1)$ LCD:

$x(x+1)(x+1) = x(x+1)^2$

② $\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

③ $5x^2+20x+6 = A(x+1)^2 + Bx(x+1) + Cx$

$$5x^2+20x+6 = A(x+1)^2 + Bx(x+1) + Cx$$

Let $x=-1$

$$5-20+6 = 0+0-C$$

$$-9 = -C$$

$$C = 9$$

Let $x=0$

$$0+0+6 = A+0+0$$

$$A = 6$$

Let $x=1$

$$5+20+6 = 6(4)+2B+9(1)$$

$$31 = 33+2B \Rightarrow B = -1$$

Final answer:

$$\frac{x^4+2x^3+6x^2+20x+6}{x^3+2x^2+x} = x + \frac{6}{x} + \frac{-1}{x+1} + \frac{9}{(x+1)^2}$$

$$\frac{7}{x^2-14x}$$

① $x(x-14)$

② $\frac{A}{x}+\frac{B}{x-14}$

③ $7=A(x-14)+Bx$

$$7 = A(x-14) + Bx$$

Let $x = 14$

$7 = 0 + 14B$

$B = \frac{1}{2}$

Let $x = 0$

$7 = -14A + 0$

$A = -\frac{1}{2}$

Final Answer: $\frac{7}{x^2-14x} = \frac{-\frac{1}{2}}{x} + \frac{\frac{1}{2}}{x-14} = \frac{-1}{2x} + \frac{1}{2(x-14)}$

$$\frac{3x^2+4x+4}{x^3+4x}$$

① $x(x^2+4)$

② $\frac{A}{x}+\frac{Bx+C}{x^2+4}$

③ $3x^2+4x+4 = A(x^2+4)+(Bx+C)x$

$$3x^2+4x+4 = A(x^2+4)+(Bx+C)x$$

$$3x^2+4x+4 = Ax^2+4A+Bx^2+Cx$$

$$3x^2+4x+4 = (A+B)x^2+Cx+4A$$

$3 = A+B$

$1+B=3$

$B=2$

$4=C$

$C=4$

$4=4A$

$A=1$

Final Answer:

$$\frac{3x^2+4x+4}{x^3+4x} = \frac{1}{x} + \frac{2x+4}{x^2+4}$$

$$\frac{8x^3+13x}{(x^2+2)^2}$$

① done $(x^2+2)^2$

② $\frac{Ax+B}{x^2+2} + \frac{Cx+D}{(x^2+2)^2}$

③ $8x^3+13x = (Ax+B)(x^2+2)+(Cx+D)(1)$

$$8x^3+13x=(Ax+B)(x^2+2)+(Cx+D)(1)$$

$$8x^3+13x=Ax^3+Bx^2+2Ax+2B+Cx+D$$

$$8x^3+13x=Ax^3+Bx^2+(2A+C)x+(2B+D)$$

$8=A$ $\quad A=8$

$B=0$

$2A+C=13$

$2(8)+C=13$

$C=-3$

$2B+D=0$

$0+D=0$

$D=0$

Final answer:

$$\frac{8x^3+13x}{(x^2+2)^2} = \frac{8x}{x^2+2} + \frac{-3x}{(x^2+2)^2}$$